DEYTRON YADROSI UCHUN NN POTENSIAL CHUQURLIGINI SONLI USULDA ANIQLASH.

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Annotatsiya: Bog'langan yadrolarning nuklon-nuklon (NN) potensial chuqurligini aniqlash bu yadrolarning to'lqin funksiyasini aniqlashda muhim ahamiyatga ega hisoblanadi. Jumladan yadroviy reaksiyalarning ro'y berish ehtimolligi, effektiv kesimi va reaksiya tezliklarini aniqlashda shu reaksiyada ishtirok etayotgan yadrolarning to'lqin funksiyasini bilish talab qilinadi. Biz bu ishda deytron yadrosining NN potensial chuqurligini (Gauss va eksponensial potensiallar uchun) "wolfram mathematica" dastur paketidan foydalangan holda sonli usulda aniqladik.

Kalit so'z: potensial chuqurlik, NN potensial, to'lqin funksiya.

Аннотация: Определение потенциальной глубины НН связанных ядер имеет важное значение для определения волновой функции этих ядер. В частности, при определении вероятности протекания ядерных реакций, эффективного сечения и скорости реакции необходимо знать волновую функцию ядер, участвующих в этой реакции. В данной работе мы определили глубину НН-потенциала ядра дейтрона (для гауссового и экспоненциального потенциалов) численно с использованием пакета программ wolfram mathematica.

Ключевые слова: глубина потенциала, потенциал NN, волновая функция.

Abstract: Determining the potential depth of NN bound nuclei is important for determining the wave function of these nuclei. In particular, when determining the probability of the occurrence of nuclear reactions, the effective cross section and the reaction rate, it is necessary to know the wave function of the nuclei participating in this reaction. In this paper, we determined the depth of the NN potential of the deuteron nucleus (for Gaussian and exponential potentials) numerically using the wolfram mathematica software package.

Keywords: potential depth, NN potential, wave function.

Kirish.

Deytron yadrosi bitta proton va bitta neytrondan iborat bog'langan sistemadir. Deytron yadrosi asosan deyteriy atomining ionlanishi natijasida yoki sekin neytronlarning proton tomoidan tutib qolishi natijasida yuzaga keladi.

Deytron yadrosining asosiy xarakteristikalari:

Spin moment $S = 1$

Orbital moment $L = 0$

Magnit momenti $\mu_{deytron} = 0.857436\mu_B$, bu yerda μ_B Bor magnetoni

Bog'lanish energiyasi $E_b = -2.2245\ MeV$

Deytron yadrosi uchun NN potensial chuqurlikni aniqlash jarayonida NN potensialning ko'rinishi muhim hisoblanadi. Ko'pgina adabiyotlarda NN potensialning ko'rinishi to'g'ri burchakli bo'lgan hol uchun hisoblashlar amalga oshirilgan[1],[2]. Masalan Masatsugu Sei Suzukining hisoblashlariga ko'ra to'g'ri burchakli shakldagi potensial uchun $V_0 = 33.73\ MeV$ ga teng bo'lganligini ko'rishimiz mumkin.

Deytron yadrosidagi proton va neytronlarning o'zaro ta'sir potensiallari ko'rinishi quyidagicha bo'lgan hollar uchun hisoblashlarni amalga oshiramiz:

a) Exponensial potensial

$$V_N(r) = -V_0 e^{-\frac{r}{R_N}} \quad (1)$$

b) Gauss potensiali

$$V_N(r) = -V_0 e^{-\frac{r^2}{R_N{}^2}} \quad (2)$$

Bu yerda V_0 NN potensial chuqurligi, R_N esa NN potensial parametri hisoblanadi.

Asosiy qism.

Ikkita bo'glangan yadrodan iborat sistemaga umumiy holda Kulon va yadro potensiali ta'sir qiladi.

$$V(r) = V_c(r) + V_N(r) \quad (3)$$

Bu yerda $V_c(r) = \frac{z_1 z_2 e^2}{4\pi\varepsilon_0 r}$ ga teng. Yadro potensiali sifatida yuqorida keltirilgan ikkita potensialni olamiz. Deytron yadrosi uchun potensial o'ra chuqurligi V_0 ni aniqlash uchun Shredenger tenglamasidan foydalanamiz:

$$-\frac{\hbar^2}{2\mu}\nabla^2\psi + V(r)\psi = E\psi \quad (4)$$

Bu yerda $\mu = \frac{m_1 m_2}{m_1 + m_2}$ keltirilgan massa bo'lib inersiya markazi sistemasiga o'tilgan. Bizning holda m_1 va m_2 mos holda proton va neytronning massasi.

Shredenger tenglamasini sferik koordinatada yechamiz chunki yadroviy potensial markaziy simmetrik potensial.

$$-\frac{\hbar^2}{2\mu}\left(\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d\psi}{dr}\right) - \frac{L(L+1)}{r^2}\psi\right) + V(r)\psi = E\psi \quad (5)$$

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d\psi}{dr}\right) - \frac{L(L+1)}{r^2}\psi + \frac{2\mu}{\hbar^2}\left(E - \frac{z_1 z_2 e^2}{4\pi\varepsilon_0 r} + V_N(r)\right)\psi = 0 \quad (6)$$

Bu tenglamada $\psi(\theta,\varphi,r) = \frac{\chi_L(r)}{r} Y_{L,m}(\theta,\varphi)$ kabi almashtirish bajaraylik.

$$\frac{d^2\chi_L(r)}{dr^2} - \frac{L(L+1)}{r^2}\chi_L(r) + \frac{2\mu}{\hbar^2}\left(E - \frac{z_1 z_2 e^2}{4\pi\varepsilon_0 r} + V_N(r)\right)\chi_L(r) = 0 \quad (7)$$

Chegaraviy shartlarni aniqlaylik. Agar $r = r_{min}$ (r_{min} nolga juda yaqin masofa) bo'lsa $k_0 = \sqrt{\frac{2\mu}{\hbar^2}(E + V_0)}$ deb belgilasak

$$\frac{d^2 U_L(r)}{dr^2} + \left(k_0^2 - \frac{L(L+1)}{r^2}\right)U_L(r) = 0 \quad (8)$$

Hosil bo'lgan tenglamada $\rho = k_0 r$ deb belgilab $U_L = \rho^{\frac{1}{2}}\phi(\rho)$ almashtirish bajarsak

$$\phi'' + \frac{1}{\rho}\phi' + \left(1 - \frac{\left(L+\frac{1}{2}\right)^2}{\rho^2}\right)\phi = 0 \quad (9)$$

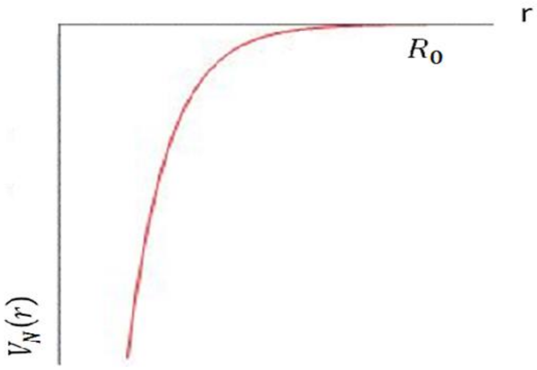
Bu tenglama sferik Bessel tenglamasidir.

U holda yechim $U_L(r) = \sqrt{\frac{\pi k_0 r}{2}} J_{L+\frac{1}{2}}(k_0 r)$ (10)

Demak yuqoridagi (7) tenglamaning radial yechimi uchun chegaraviy shartlarni yozish mumkin:

$\chi_L(r)|_{r=r_{min}} = U_L(r_{min})$, $\chi_L{}'(r)|_{r=r_{min}} = k_0 U_L{}'(r_{min})$ (11)

Shunday bir $r = R_0$ masofa borki bu nuqtadan katta masofalarda yadro potensiali nolga teng(6-rasm).



1-rasm. Ikkita nuklon yadro potensialining masofaga bog'liqligi taxminiy ko'rinishi.
Ya'ni bu nuqtadan tashqarida Kulon maydoni muhim rol o'ynaydi. Shu nuqtadan tashqarida (7) tenglama yechimi qanday korinishda bo'lishini aniqlaylik.
Biz ko'rayotgan sohada (7) tenglamani ko'rinishi quyidagicha bo'ladi:

$$\frac{d^2\xi_L(r)}{dr^2}-\frac{L(L+1)}{r^2}\xi_L(r)+\frac{2\mu}{\hbar^2}\left(E-\frac{z_1z_2e^2}{4\pi\varepsilon_0 r}\right)\xi_L(r)=0 \qquad (12)$$

$k=\sqrt{\frac{2\mu E}{\hbar^2}}$ deb belgilash kiritamiz.

$$\frac{d^2\xi_L(r)}{dr^2}-\frac{L(L+1)}{r^2}\xi_L(r)+k^2\xi_L(r)-\frac{2\mu z_1z_2e^2}{\hbar^2 4\pi\varepsilon_0}\frac{\xi_L(r)}{r}=0 \qquad (13)$$

Bu tenglamani $4k^2$ ga bo'lib yuboraylik:

$$\frac{d^2\xi_L(r)}{d(2kr)^2}-\frac{L(L+1)}{(2kr)^2}\xi_L(r)+\frac{1}{4}\xi_L(r)-\frac{\mu z_1z_2e^2}{k\hbar^2 4\pi\varepsilon_0}\frac{\xi_L(r)}{2kr}=0 \qquad (14)$$

Agar $v=\frac{\hbar k}{\mu}$ ekanligini e'tiborga olsak

$$\frac{d^2\xi_L(r)}{d(2kr)^2}-\frac{L(L+1)}{(2kr)^2}\xi_L(r)+\frac{1}{4}\xi_L(r)-\frac{z_1z_2e^2}{4\pi\varepsilon_0\hbar v}\frac{\xi_L(r)}{2kr}=0 \qquad (15)$$

Zommerfeld parametri $\eta_0=\frac{z_1z_2e^2}{4\pi\varepsilon_0\hbar v}$ ni kiritsak tenglama quyidagicha ko'rinishga keladi:

$$\frac{d^2\xi_L(r)}{d(2kr)^2}-\frac{L(L+1)}{(2kr)^2}\xi_L(r)+\frac{1}{4}\xi_L(r)-\eta_0\frac{\xi_L(r)}{2kr}=0 \qquad (16)$$

$\rho=2kr$ deb belgilash kiritamiz:

$$\frac{d^2\xi_L(r)}{d(\rho)^2}-\frac{L(L+1)}{(\rho)^2}\xi_L(r)+\frac{1}{4}\xi_L(r)-\eta_0\frac{\xi_L(r)}{\rho}=0 \qquad (17)$$

Bu tenglamani $m=L+\frac{1}{2}$ belgilash kiritish orqali ko'rinishini almashtiraylik. Natijada tenglama quyidagicha bo'ladi.

$$\left(\frac{d^2}{d\rho^2}-\frac{1}{4}+\frac{\eta_0}{\rho}+\frac{\frac{1}{4}-m^2}{\rho^2}\right)\xi_L(r)=0 \qquad (18)$$

Bu tenglama Witteker tenglamasining o'zidir. Shu bois yechim

$$\xi_L(r)=C_1W_{-\eta_0,L+\frac{1}{2}}(2kr)+C_2W_{\eta_0,L+\frac{1}{2}}(-2kr) \qquad (19)$$

ko'rinishda bo'ladi. Bu yerda $W_{-\eta_0,L+\frac{1}{2}}(2kr)$ va $W_{\eta_0,L+\frac{1}{2}}(-2kr)$ lar Witteker funksiyasi deyiladi. Bu funksiya (1) tenglamaning katta masofalarda asimptotikasining o'zginasidir, ya'ni

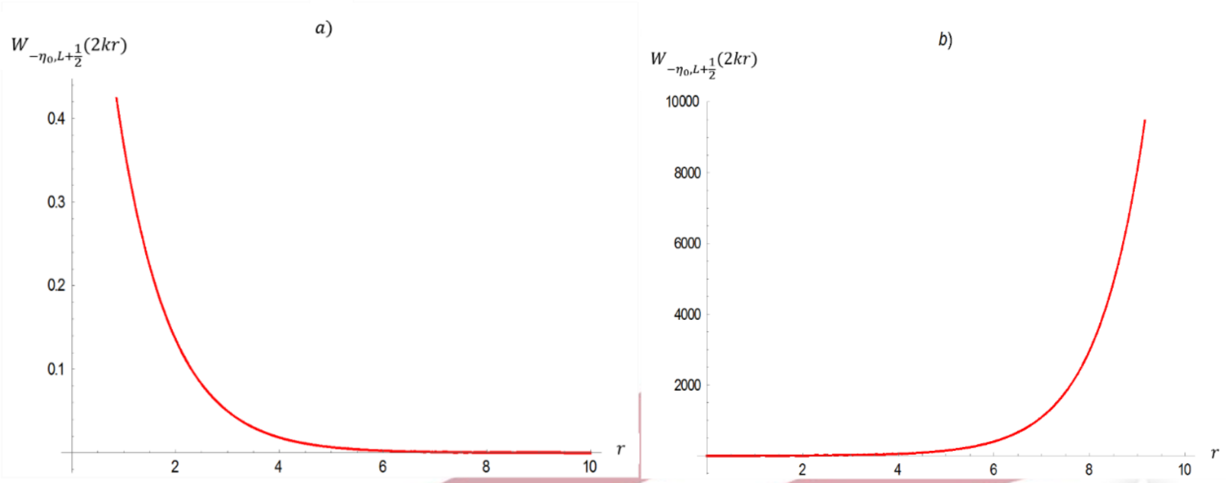
$$\chi_L{}^{as}(r)=\xi_L(r)=C_1W_{-\eta_0,L+\frac{1}{2}}(2kr)+C_2W_{\eta_0,L+\frac{1}{2}}(-2kr) \qquad (20)$$

Witteker funksiyasini katta masofalarda asimptotikasi quyidagi ko'rinishga ega:

$$W_{-\eta_0,L+\frac{1}{2}}(2kr)\rightarrow\frac{e^{-kr}}{(2kr)^{\eta_0}} \qquad (21)$$

$$W_{\eta_0,L+\frac{1}{2}}(-2kr)\rightarrow e^{kr}(2kr)^{\eta_0} \qquad (22)$$

Bu funksiyalarni grafigi quyidagicha ko'rinishga ega (7-rasm).



2-rasm. Witteker funksiyasi asimptotikasining masofaga bog'lanish grafigi.

2b grafikdan ko'rinib turibdiki katta masofalarda $W_{\eta_0,L+\frac{1}{2}}(-2kr)$ funksiya cheksizga intiladi. Kvant mexanikasining fundamental qonunlariga asosan to'lqin funksiya chekli bo'lishi kerak.

Bir-biriga juda yaqin ikkita shunday r_1 va r_2 nuqtalarni olamizki, (bu nuqtalarda $r_1 > R_0$ hamda $r_2 > R_0$ shart bajarilib, asosiy rolni Kulon potensiali bajaradi) unda (7) tenglamaning yechimi

$$\chi_L(r_1) = C_1 W_{-\eta_0,L+\frac{1}{2}}(2kr_1) + C_2 W_{\eta_0,L+\frac{1}{2}}(-2kr_1) \quad (23a)$$

$$\chi_L(r_2) = C_1 W_{-\eta_0,L+\frac{1}{2}}(2kr_2) + C_2 W_{\eta_0,L+\frac{1}{2}}(-2kr_2) \quad (23b)$$

ko'rinishda bo'ladi bo'ladi. Bulardan $d = \frac{C_2}{C_1}$ nisbatni aniqlaymiz.

$$\frac{\chi_L(r_1)}{\chi_L(r_2)} = \frac{W_{-\eta_0,L+\frac{1}{2}}(2kr_1) + dW_{\eta_0,L+\frac{1}{2}}(-2kr_1)}{W_{-\eta_0,L+\frac{1}{2}}(2kr_2) + dW_{\eta_0,L+\frac{1}{2}}(-2kr_2)} \quad (24)$$

$$d = \frac{\chi_L(r_1) W_{-\eta_0,L+\frac{1}{2}}(2kr_2) - \chi_L(r_2) W_{-\eta_0,L+\frac{1}{2}}(2kr_1)}{\chi_L(r_2) W_{\eta_0,L+\frac{1}{2}}(-2kr_1) - \chi_L(r_1) W_{\eta_0,L+\frac{1}{2}}(-2kr_2)} \quad (25)$$

Chunki to'lqin funksiya chekli bo'lishi uchun bu kattalik nolga juda yaqin bo'lishi kerak. Endi agarda biz potensial chuqurlik V_0ni qiymatini biror boshlang'ich qiymatdan boshlab o'zgartirib borib, (7) tenglamaning yechimi $\chi_L(r)$ ni r_1 va r_2 nuqtalarda sonli usulda aniqlab borsak, (25) ifodadagi d ning qiymati ham o'zgarib boradi va qaysidir nuqtada d nolga juda yaqin bo'ladi. Shu nuqtadagi V_0 ning qiymati biz izlayotgan potensial chuqurlikni beradi. Wolfram mathematica dastur paketidan foydalangan holda hisoblashlar amalga oshirilib quyidagi natijalar olindi:

Potensiallar / Parametrlar	Eksponensial potensial	Gauss potensiali
R_N	$0.683\ fm$	$1.65\ fm$
V_0	$184.08\ MeV$	$60.5713\ MeV$

Xulosa.

Deytron yadrosi uchun NN potensial chuqurlik eksponensial va Gauss potensiali uchun hisoblandi. Hisoblash wolfram mathematica dastur paketi yordamida sonli usulda olib borildi. Deytron yadrosi bo'g'langan yadro bo'lganligi uchun to'lqin funksiya katta masofalarda chegaralangan bo'lishi lozim. Shu bois potensial chuqurlikni aniqlash jarayonida Shredenger tenglamasining asimptotik yechimidan foydalandik.

Foydalanilgan adabiyotlar.

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