CHIZIQLI ALGEBRA VA ANALITIK GEOMETRIYANING TANLANMA MASALALARI

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Annotatsiya
Ushbu maqolada chiziqli algebra va analitik geometriyaning qiziqarli tanlanma masalalari va ularning yechimlari keltirilgan bo‘lib, bunday masalalarni yechish jarayonida talabalarning avvalambor, fanga qiziqishlari ortishlari, egallagan bilimlarini bir-biriga bog‘lay olishlari hamda boshqa shu kabi masalalarni yechish uchun yetarli ko‘nikma va malakalarga ega bo‘lishlari ko‘zda tutilgan.

Kalit so‘zlar: Chiziqli algebra, fibonachchi sonlari, determinant, determinantning xossalari, rekkurent formula, tenglama, bir jinsli tenglamalar sistemasi.

Chiziqli algebra va analitik geometriya amaliy matematika yo‘nalishi talabalarining mutaxassislik fani bo‘lib, fanning dastlabki modullari matritsa va determinantlar hisoblanadi. Determinantlar va ularning xossalariga oid ko‘plab misol va masalalar mavjud bo‘lib, biz quyida shunday masalalarning tanlanganlarini ko‘rib chiqamiz.

1-masala. Fibonachchi sonlari deb 1, 2 dan boshlanuvchi shunday sonlar qatoriga aytiladiki, har bir keyingi had oldingi ikkita hadning yig‘indisiga teng. 1,2,3,5,8,13,... Fibonachchi sonlarining n - hadi quyidagi determinantga tengligini isbotlang:

$$\begin{vmatrix} 1 & 1 & 0 & 0 & \dots & 0 & 0 \\ -1 & 1 & 1 & 0 & \dots & 0 & 0 \\ 0 & -1 & 1 & 1 & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & 0 & \dots & -1 & 1 \end{vmatrix}$$

Yechimi: Buning uchun determinantni biror D_n deb belgilab olib, uni 1-va 2-satrlari bo‘yicha yoyamiz:

$$D_n = \begin{vmatrix} 1 & 1 & 0 & 0 & \dots & 0 & 0 \\ -1 & 1 & 1 & 0 & \dots & 0 & 0 \\ 0 & -1 & 1 & 1 & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & 0 & \dots & -1 & 1 \end{vmatrix} = D_{n-1} + \begin{vmatrix} 1 & 0 & 0 & \dots & 0 & 0 \\ -1 & 1 & 1 & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & -1 & 1 \end{vmatrix}$$

hosil bo‘lgan (n - 1) - tartibli determinantni ham 1-satr bo‘yicha yoysak,

$D_n = D_{n-1} + D_{n-2}$ tenglikka ega bo‘lamiz. Bu Fibonachchi sonlari ketma-ketligining rekkurent formulasini ifodalaydi. Endi buni tekshirib ko‘rish qoldi.

$$D_1 = 1, \quad D_2 = \begin{vmatrix} 1 & 1 \\ -1 & 1 \end{vmatrix} = 2, \quad D_3 = \begin{vmatrix} 1 & 1 & 0 \\ -1 & 1 & 1 \\ 0 & -1 & 1 \end{vmatrix} = 3.$$

bu esa rekkurent formulani qanoatlantiradi. Demak, determinantning n - hadi Fibonachchi sonlari ketma-ketligining n - hadiga teng ekan.

2-masala. Tenglamani yeching.

$$\begin{vmatrix} x & c_1 & c_2 & \dots & c_n \\ c_1 & x & c_2 & \dots & c_n \\ \dots & \dots & \dots & \dots & \dots \\ c_1 & c_2 & c_3 & \dots & x \end{vmatrix}$$

Yechimi: Dastlab, 2-ustundan boshlab barcha ustun elementlarini 1-ustun elementlariga qo'shib chiqamiz:

$$\begin{vmatrix} x+c_1+c_2+\dots+c_n & c_1 & c_2 & \dots & c_n \\ x+c_1+c_2+\dots+c_n & x & c_2 & \dots & c_n \\ \dots & \dots & \dots & \dots & \dots \\ x+c_1+c_2+\dots+c_n & c_2 & c_3 & \dots & x \end{vmatrix} = 0$$

$$\Rightarrow \quad (x+c_1+c_2+\dots+c_n)\begin{vmatrix} 1 & c_1 & c_2 & \dots & c_n \\ 1 & x & c_2 & \dots & c_n \\ \dots & \dots & \dots & \dots & \dots \\ 1 & c_2 & c_3 & \dots & x \end{vmatrix}$$

endi 1-satr elementlarini -1 ga ko'paytirib, 2-satr elementlariga, 2-satr elementlarini -1 ga ko'paytirib 3-satr elementlariga qo'shamiz va hokazo. (n-1) -satr elementlarini -1 ga ko'paytirib, n-satr elementlariga qo'shgandan keyin quyidagi ifodani hosil qilamiz:

$$(x+c_1+c_2+\dots+c_n)\begin{vmatrix} 1 & c_1 & c_2 & \dots & c_n \\ 0 & x-c_1 & 0 & \dots & 0 \\ 0 & 0 & x-c_2 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & x-c_n \end{vmatrix} = 0$$

Determinantni xossasidan foydalanib hisoblasak,

$$(x+c_1+c_2+\dots+c_n)(x-c_1)(x-c_2)\dots(x-c_n) = 0$$

hosil qilamiz. Bundan esa

$$x_1 = -(c_1+c_2+\dots+c_n), \quad x_2 = c_1, x_3 = c_2, \dots x_{n+1} = c_n$$

yechimlarni olamiz.

3-masala. a_{ij} $(i,j=\overline{1,n})$ koeffitsiyentlar butun son bo'lsa, quyidagi tenglamalar sistemasi yagona yechimga ega bo'lishini ko'rsating:

$$\begin{cases} \frac{1}{2}x_1 = a_{11}x_1 + \dots + a_{1n}x_n \\ \frac{1}{2}x_2 = a_{21}x_1 + \dots + a_{2n}x_n \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ \frac{1}{2}x_n = a_{n1}x_1 + \dots + a_{nn}x_n \end{cases}$$

Yechimi: Dastlab, Sistemada tenglikning chap tarafidagi ifodalarni o'ng tarafga o'tkazib soddalashtirib olamiz. Natijada bir jinsli tenglamalar sistemasi hosil bo'ladi. So'ngra, uning quyidagi determinantini tuzamiz:

$$D = \begin{vmatrix} a_{11}-\frac{1}{2} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22}-\frac{1}{2} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nn}-\frac{1}{2} \end{vmatrix}$$

Agar biz

$$P(\lambda) = \begin{vmatrix} a_{11}-\lambda & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22}-\lambda & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nn}-\lambda \end{vmatrix}$$

Belgilash kiritsak, u holda $D = P(\frac{1}{2})$ bo‘ladi. Ikkinchi tomondan, $P(\lambda) == (-1)^n\lambda^n + b_1\lambda^{n-1} + \dots + b_n$ ko‘rinishda bo‘ladi, bu yerda b_i −butun son va $i = \overline{1,n}$. Agarda $P\left(\frac{1}{2}\right) = 0$ bo‘lsa, u holda $(-1)^n\frac{1}{2^n} + b_1\frac{1}{2^{n-1}} + \dots + b_n = 0$

Va bu oxirgi tenglikni ikkala tarafini ham 2^n ga ko‘paytirsak, $(-1)^n + 2b_1 + 2^2b_2 + \dots + 2^nb_n = 0$ bo‘ladi. Ushbu $2b_1 + 2^2b_2 + \dots + 2^nb_n = N$ belgilash kiritsak, u holda $(-1)^n + 2N = 0$ ega bo‘lamiz. Bunday bo‘lishi mumkin emas, chunki N butun. Demak, $D = P(\frac{1}{2}) \neq 0$ ekan, u holda sistema yagona yechimga ega va bu yechim $x_1 = x_2 = \dots = x_n = 0$ bo‘ladi.

Shu va shu kabi masalalarni yechish jarayonida talabalar o‘zlari egallagan bilimlarni intuitiv tarzda qo‘llay olishga hamda murakkab masalalarni yechishga dastlabki ko‘nikma va malakalarni egallashga o‘rganishadi.

Foydalanilgan adabiyotlar ro‘yxati

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